

A sample space is a set of all possible outcomes for an activity or experiment. (dual)

Ex. What is the sample space of the fingers on your right hand?  
(The denominator in a probability)

{ thumb, index, middle, ring, pinky }

What is the sample space of this spinner?



{ Red, yellow, blue }

Event - in a probability experiment, a subset of the sample space.

↳ The numerator in a probability.

What would classify as an event when the sample space is your hands?

Ex.1 { thumb }      Ex.2 { ring }      Ex.3 { ring, middle, index }

What would be an event for the spinner?

Ex.1 { Red }      Ex.2 { Yellow }

(dual)

The probability of an event, A, occurring is represented as  $P(A)$ .

Probability is expressed as a number from 0 to 1 that shows how likely an event is to occur.

It can be written as a fraction, a decimal, or a percent and is found with the following ratio.

because probability cannot be more than 100%

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}} = \frac{\text{Event}}{\text{Sample Space}}$$

### Key Idea

Write each of the following as a percent, decimal and fraction.

1.  $2/3 \approx 0.667 = 66.7\%$

2.  $0.4 = 40\% = \frac{2}{5}$

3.  $2\% = 0.02 = \frac{1}{50}$

4.  $3/7 \approx 0.429 = 42.9\%$

5.  $0.067 = 6.7\% = \frac{67}{100}$

6.  $95.67\% = 0.9567 = \frac{9567}{10000}$

Examples of Probability:

ex.1 Rolling a 2 on a 6 sided die  $\{1, 2, 3, 4, 5, 6\}$   
 $P(2) = \frac{1}{6}$

Sample space  
 ↑  
 Event

ex. 2 Rolling an even number on a 12 sided die.

die  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$   $P(\text{Even}) = \frac{6}{12} = \frac{1}{2}$

Sample space    Event

ex. 3 Drawing a four out of a deck of cards.

$$P(\text{four}) = \frac{4}{52} = \frac{1}{13}$$

Mutually Exclusive events (A.K.A. disjoint or independent) - events that cannot occur at the same time. <sup>(dual)</sup>

Examples:

Turning left and turning right

Getting heads and tails

Drawing a King and Ace

Non-examples (not mutually exclusive/  
dependent events)

Turning left and scratching your head

Drawing a king and a heart

Determine whether these events are mutually exclusive. <sup>(dual)</sup>

1) Roll a die: get an even number and get an number less than 3 *Not mutually exclusive*

2) Roll a die: get a prime number and get an odd number *Not mutually exclusive*

3) Roll a die: get a number greater than 3 and get a number less than 3.

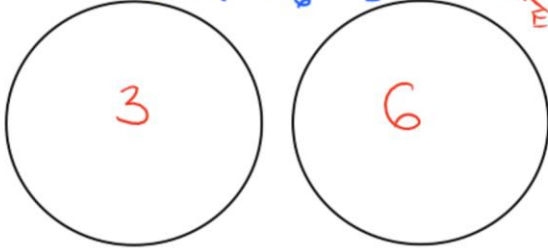
4) Select a student in the classroom: *mutually exclusive*  
Student has blond hair and blue eyes.

5) Select a student at UGA: Student is a sophomore and the student is a business major *Not mutually exclusive*

6) Select any high school course: The course is calculus and the course is English. *mutually exclusive*

**Addition Rule 1:** When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probabilities of each event.  $P(A \text{ or } B) = P(A) + P(B)$

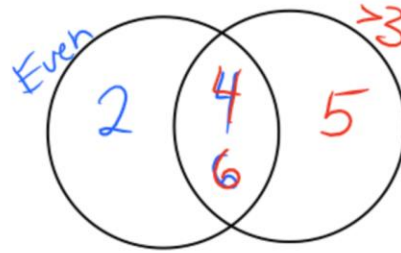
Ex. Rolling a 6 sided die and getting a 3 or 6.  
 $P(3 \text{ or } 6) = P(3) + P(6)$   
 $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$



**Addition Rule 2:** When two events, A and B, are non-mutually exclusive, the probability that A or B will occur is :

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Ex. Rolling a 6 sided die and getting an even number or a number greater than 3.  
 $P(\text{Even or } >3) = P(\text{Even}) + P(>3) - P(\text{Even and } >3)$   
 $= \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$



Ex 1. In a math class of 30 students, 17 are boys and 13 girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student.

$$P(G \text{ or } A) = P(G) + P(A) - P(G \text{ and } A)$$

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30} = \frac{17}{30}$$

$$P(G) = \frac{13}{30}$$

$$P(A) = \frac{9}{30}$$

$$P(G \text{ and } A) = \frac{5}{30}$$

Ex. 2 On New year's eve, the probability of a person having a car accident is 0.09. The probability of a person driving while intoxicated is 0.32 and the probability of a person have a car accident while intoxicated is 0.15. What is the probability of a person driving while intoxicated or having a car accident?

$$P(I \text{ or } A) = P(I) + P(A) - P(I \text{ and } A)$$

$$P(I) = 0.32 \quad P(A) = 0.09 \quad P(I \text{ and } A) = 0.15$$

$$P(I \text{ or } A) = 0.32 + 0.09 - 0.15$$

$$= 0.26$$

## Multiplication Rule *key word "and"*

$$P(A \text{ and } B) = P(A) * P(B)$$

Used to find the probability of independent or disjoint events happening one right after another.

EXAMPLE 1: Select two cards from the standard deck of 52 cards with replacement. Find the probability of selecting two kings.

$$\begin{aligned} P(K \text{ and } K) &= P(K) \cdot P(K) \\ &= \frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2704} = \frac{1}{169} \end{aligned}$$

EXAMPLE 2: Suppose you toss a coin and then roll a dice. What is the probability of obtaining a tail and then rolling a 5

$$\begin{aligned} P(\text{Tail and } 5) &= P(\text{Tail}) \cdot P(5) \\ &= \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} \end{aligned}$$

Consider the following two problems:

(dual)

(1) **Select 2 cards from a standard deck of 52 cards with replacement. What is the probability of obtaining two kings?**

$$P(K \text{ and } K) = \frac{1}{169}$$

(2) Select 2 cards from a standard deck of 52 cards without replacement. What is the probability of obtaining two kings?

$$\begin{aligned} P(K \text{ and } K) &= P(K) \cdot P(K|K) \\ &= \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \end{aligned}$$

Do these have the same probability?

No

EXAMPLE 3: Select two cards from the standard deck of 52 cards without replacement. Find the probability of selecting two kings

From the previous slide

$$P(K \text{ and } K) = \frac{1}{221}$$

A committee consists of four women and three men. The committee will randomly select two people to attend a conference in Hawaii. Find the probability that both are women

$$P(W \text{ and } W) = P(W) \cdot P(W|W)$$

↙ given

$$P(W \text{ and } W) = \frac{4}{7} \cdot \frac{3}{6} = \frac{12}{42} = \frac{4}{21}$$

### Conditional probability Formula

$$P(\text{A and B}) = P(A) \times P(B|A)$$

"Probability Of"
"Given"

Event A
Event B

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

$$P(B \text{ and } W) = 0.34 \quad P(B) = 0.47$$

$$P(W|B) = ?$$

$$P(W|B) = \frac{P(B \text{ and } W)}{P(B)} = \frac{0.34}{0.47} \approx 0.723$$

The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

$$P(\text{Fri. and Absent}) = 0.03 \quad P(\text{Fri}) = 0.2$$

$$P(\text{Absent} \mid \text{Fri.}) = ?$$

$$\begin{aligned} P(\text{Absent} \mid \text{Fri.}) &= \frac{P(\text{Fri. and Absent})}{P(\text{Fri})} \\ &= \frac{0.03}{0.2} = 0.15 \end{aligned}$$





