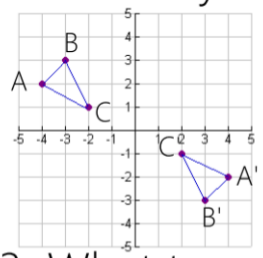
Warm-Up

Congruence

1. What does it mean to say two shapes are congruent?

The sides and angles are congruent.

2. Identify the transformation shown.



Rotation of 180°.

3. What transformations keep angle measure, and side lengths? Translation, Rotation, and Reflection

Which do not?

Dilation

What are these types of transformations called?

Rigid motions

Vocabulary

Congruence

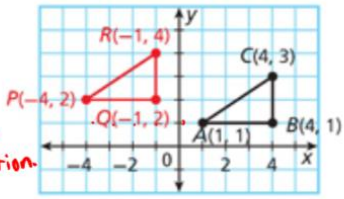
Congruence - have the same size and shape. Two figures are defined to be congruent if there is a sequence of rigid motions that maps one to the other.

Rigid motion - a transformation that preserves side lengths and angles

Rigid Motions create Congruent Figures

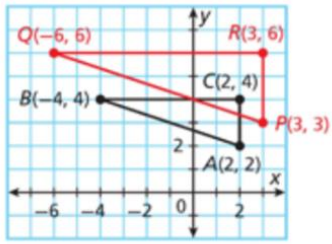
Determine whether the polygons with the given vertices are congruent.

A $A(1, 1), B(4, 1), C(4, 3)$
$P(-4, 2), Q(-1, 2), R(-1, 4)$



ΔABC can be mapped to ΔPQR by a translation $(x,y) \rightarrow (x-5, y+1)$. A translation is a rigid motion.
Thus, $\Delta ABC \cong \Delta PQR$

B $A(2, 2), B(-4, 4), C(2, 4)$
$P(3, 3), Q(-6, 6), R(3, 6)$

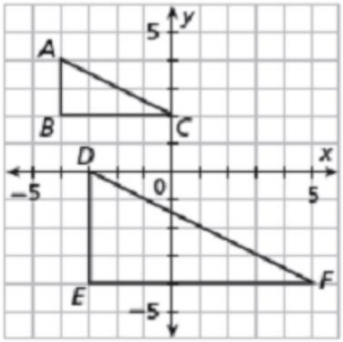


ΔABC can be mapped to ΔPQR by a dilation with a scale factor of 2.
A dilation is not a rigid motion.
Thus, $\Delta ABC \neq \Delta PQR$

Use the definition of congruence in terms of rigid motions to determine whether the two figures are congruent and explain your answer.

Congruence

1.

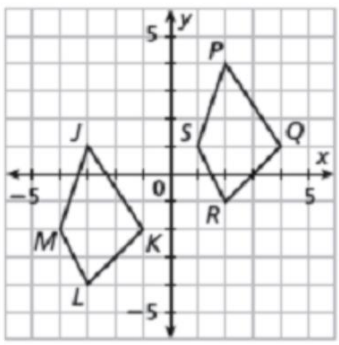


ΔABC can be mapped to ΔDEF by a dilation with scale factor of 2.
A dilation is not a rigid motion. Thus, $\Delta ABC \neq \Delta DEF$

Use the definition of congruence in terms of rigid motions to determine whether the two figures are congruent and explain your answer.

Congruence

2.

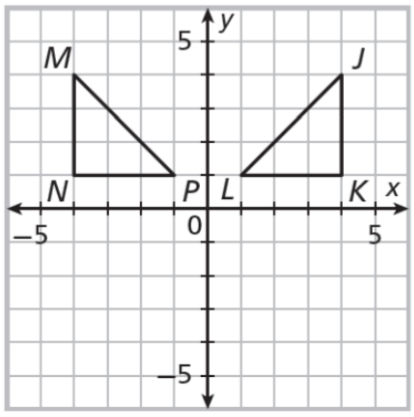


JKLM can be mapped to PQRS by a translation of $(x,y) \rightarrow (x+5, y+3)$.
A translation is a rigid motion.
Thus, JKLM $\cong$ PQRS.

Extra example

Use the definition of congruence in terms of rigid motions to determine whether the two figures are congruent and explain your answer.

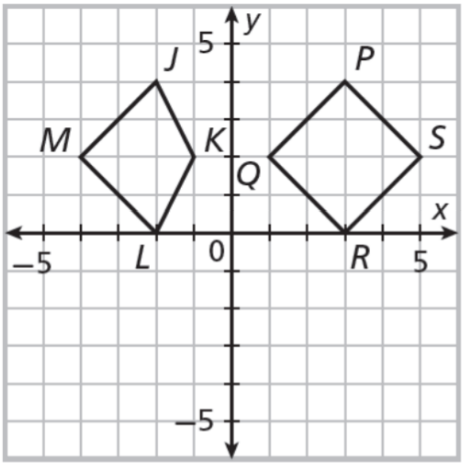
A.



$\triangle JKL$ can be mapped to $\triangle MNP$ by a reflection over the y-axis. A reflection is a rigid motion. Thus, $\triangle JKL \cong \triangle MNP$.

Extra Example

B.



JKLM cannot be mapped to PSRQ by a transformation. This means the figure cannot be mapped by a rigid motion. Thus, JKLM $\ncong$ PSRQ.

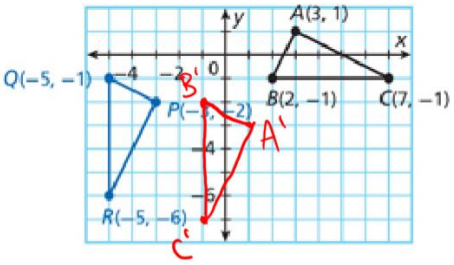
Is congruence still preserved when doing more than one rigid motion?

Yes, it is a sequence of rigid motions.

Congruence

Prove that the polygons with the given vertices are congruent

$A(3, 1), B(2, -1), C(7, -1)$
$P(-3, -2), Q(-5, -1), R(-5, -6)$

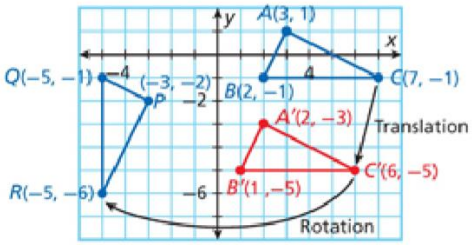


Is there any single rigid motion that would move ΔABC onto ΔPQR?

No

Could we do more than one?

Rotation of 90° cw
then translation up 1 Left 4



Congruence

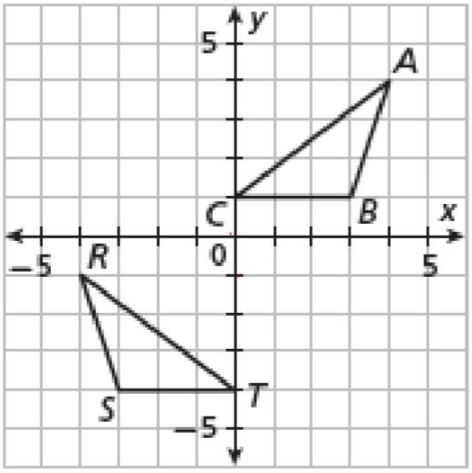
Find a sequence of rigid motions that maps one figure to the other. Write the sequence in coordinate notation

How could you map △ABC to △RST ?

ΔABC can be mapped to Δ RST by a reflection over the y-axis, then a translation down 5.

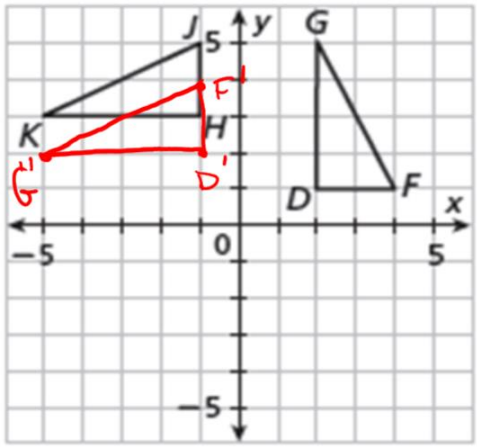
A reflection and translation are a sequence of rigid motions.

Thus, ΔABC ≅ ΔRST.



Find a sequence of rigid motions that maps one figure to the other. Write the sequence in coordinate notation.

How could you map △DFG to △HJK?



△DFG can be mapped to △HJK by a rotation of 90° CCW, then a translation up 1.

A rotation and translation are a sequence of rigid motions.

Thus △DFG ≅ △HJK