EOC Review

Name _____ Date _____ Block _____

What have we learned so far?

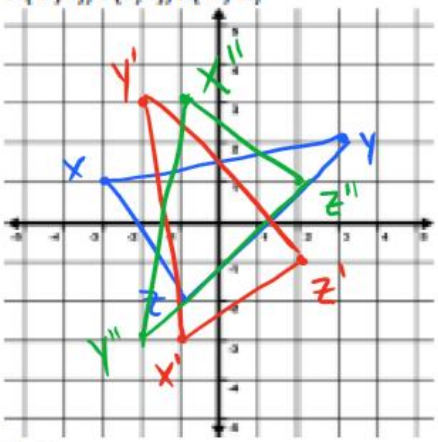
Transformations

1. Name the 4 types of transformations.
   Rotation, reflection, translation, dilation

2. Which transformation only creates similar figures and why?
   Dilations, because a dilation preserves shape, but changes the size. This is what it means to be similar.

3. What transformation are rigid motions?
   Rotation, Reflection, translation.

4. **Rotate the figure with the given vertices 90° Counter Clockwise about the origin, then reflect it over the y-axis.**
   X(-3,1), Y(3,2), Z(-1,-2)



What is the coordinate notation for the transformations given?

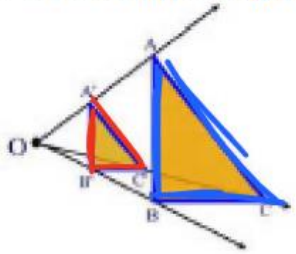
$(a,b) \rightarrow (-b,a) \rightarrow (-b,-a)$

$(a,b) \rightarrow (-b,a)$

Similarity

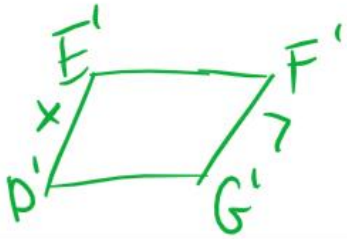
1. In terms of dilations, define the following terms in your own words.
   a. preimage – The original figure
   b. image - The figure after a transformation is done
   c. scale factor - image/preimage

2. Identify the preimage, image, and provide a possible scale factor for the following figures.



   a. preimage - ABC
   b. image - A'B'C'
   c. Scale factor - Reduction $k = \frac{1}{3}$

3. Given that figure DEFG and figure D'E'F'G' are similar and that $\overline{FG}$ is 2, $\overline{DE}$ is 5, and $\overline{F'G'}$ is 7 find the scale factor and the length of $\overline{D'E'}$.



Similar figures have proportional sides.

$\frac{7}{2} = \frac{x}{5} \rightarrow 2x = 35$

$x = 17.5$

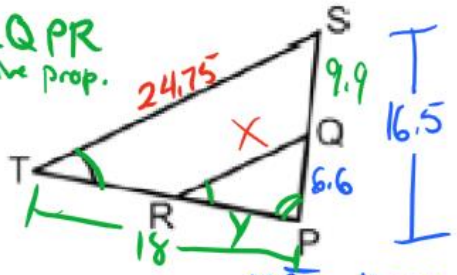
EOC Review

Triangle Similarity

1. Decide if the following triangles are similar. State the postulate or theorem that you could use to show that.

$\Delta SPT \sim \Delta QPR$ by AA. $\angle STP \cong \angle QRP$ (given) + $\angle SPT \cong \angle QPR$ (Reflexive prop.)

2. If $\overline{ST}$ is 24.75, $\overline{SP}$ is 16.5, and $\overline{SQ}$ is 9.9, what is the length of $\overline{QR}$?



$\frac{x}{24.75} \times \frac{6.6}{16.5}$ $\quad 16.5x = 163.35$ $\quad \frac{16.5x}{16.5} = \frac{163.35}{16.5}$ $\quad \overline{RQ} = 9.9$

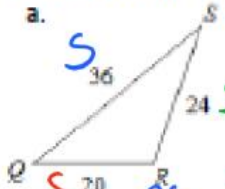
$m\overline{QP} = 16.5 - 9.9 = 6.6$

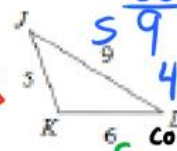
3. What is the length of $\overline{RP}$ if $\overline{TP}$ is 18?

$\frac{y}{18} \times \frac{6.6}{16.5}$ $\quad 16.5y = 118.8$ $\quad y = 7.2$ $\quad \overline{RP} = 7.2$

4. Provide evidence to show that the following triangles are similar or not. Write a similarity statement if they are.

a.





S S S

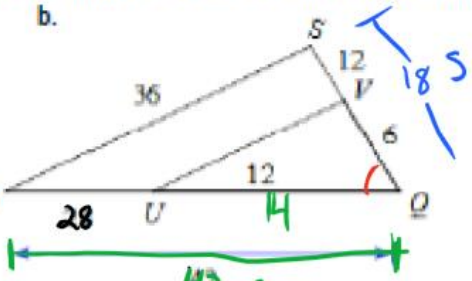
$\frac{36}{9} = \frac{24}{6} = \frac{20}{5}$

$4 = 4 = 4$

corr. sides prop.

$\Delta QRS \cong \Delta JKL$ by SSS

b.



S S

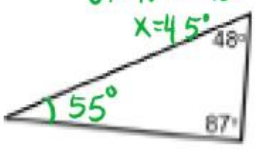
$\frac{42}{14} = \frac{18}{6}$

$3 = 3$ corr. sides prop.

$\angle VQU \cong \angle SQR$ by reflexive prop.

$\Delta SQR \cong \Delta VQU$ by SAS

c.



$87 + 48 + x = 180$

$x = 45°$

$87 + 42 + x = 180$

$x = 51°$

These triangles are not similar by AA because it only has one pair of $\cong$ angles.

Congruence as a Rigid Motion

1. A figure has the coordinates A(-1, 3), B(2, -1), C (2, -4). Apply the given transformation, identify the transformation(s), and explain if the image produced is congruent to the preimage.

a. $(x, y) \rightarrow (3x, 3y)$

$A'(-3, 9)$ $B'(6, -3)$ $C'(6, -12)$

Dilation by scale factor of 3.

$ABC \ncong A'B'C'$ because $A'B'C'$ is larger than ABC

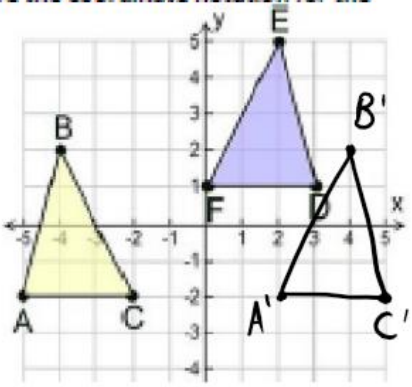
b. $(x, y) \rightarrow (-y, -x)$

$A'(-3, 1)$ $B'(1, -2)$ $C'(4, -2)$

Reflection over $y = -x$

$ABC \cong A'B'C'$ because a reflection is a rigid motion. Rigid motions create $\cong$ figures.

2. For the following figures state if they are congruent and how you know. Then give the coordinate notation for the transformations used to map one onto the other.



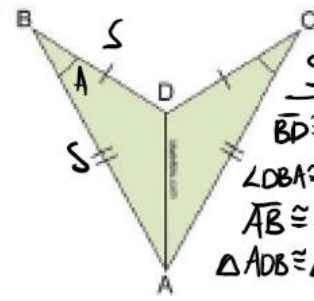
ABC can be mapped onto DEF by a reflection over the y-axis then a translation left 2 and up 3. A reflection then translation is a sequence of rigid motions. Rigid motions create congruent figures. Thus, $\Delta ABC \cong \Delta DEF$.

EOC Review

Proving Triangles Congruent

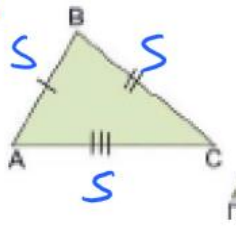
1. For the following triangles decide which triangle postulate, if any, can prove the triangles congruent. Then prove them to be congruent or state no conclusion.

a.



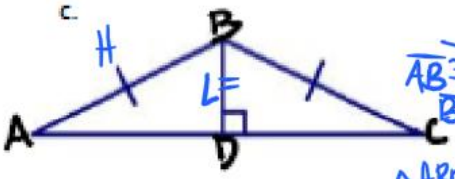
S	R
$\overline{BD} \cong \overline{CD}$	Given
$\angle DBA \cong \angle DCA$	Given
$\overline{AB} \cong \overline{AC}$	Given
$\triangle ADB \cong \triangle ADC$	SAS

b.



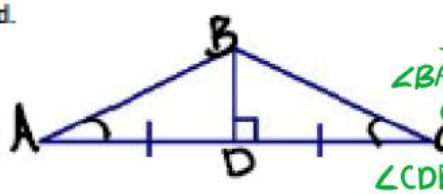
S	R
$\overline{AB} \cong \overline{DE}$	Given
$\overline{BC} \cong \overline{EF}$	Given
$\overline{CA} \cong \overline{FD}$	Given
$\triangle ABC \cong \triangle DEF$	SSS

c.



S	R
$\overline{AB} \cong \overline{CB}$	Given
$\overline{BD} \cong \overline{BD}$	Reflexive Prop.
$\triangle ABD \cong \triangle CBD$	HL

d.



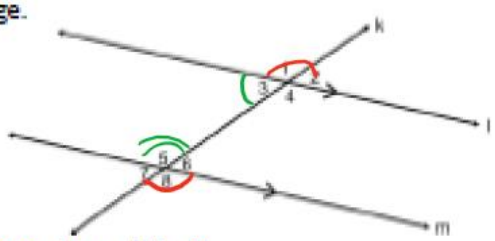
S	R
$\angle BAD \cong \angle BCD$	Given
$\overline{CD} \cong \overline{AD}$	Given
$\angle CDB \cong \angle ADB$	Given
$\triangle ABD \cong \triangle CBD$	ASA

e. In the above problem, problem letter a. has two possible ways to show congruence, what is the second way?

By SSS using $\overline{DA} \cong \overline{DA}$ by the reflexive prop.

Proving Parallel Line Properties

1. Prove the following parallel line properties to be true using the given image.



a. Given $\overleftrightarrow{l} \parallel \overleftrightarrow{m}$, prove that $\angle 3$ is supplementary to $\angle 5$.

Statement	Reason
$l \parallel m$	Given
$\angle 3 \cong \angle 7$	Corr. $\angle$'s Post.
$\angle 7$ and $\angle 5$ are linear pairs	Def. of Linear Pairs
$m\angle 7 + m\angle 5 = 180$	Linear Pairs conj.
$m\angle 3 = m\angle 7$	Def. of $\cong$
$m\angle 3 + m\angle 5 = 180°$	Substitution
$\angle 3$ and $\angle 5$ are Supplementary	Def. of Supp.

b. Given $\overleftrightarrow{l} \parallel \overleftrightarrow{m}$, prove that $\angle 1 \cong \angle 8$.

Statement	Reason
$l \parallel m$	Given
$\angle 1 \cong \angle 5$	Corr. $\angle$'s Post
$\angle 5 \cong \angle 8$	Vertical $\angle$'s Thm.
$\angle 1 \cong \angle 8$	Transitive Prop.

EOC Review

Triangle Theorems

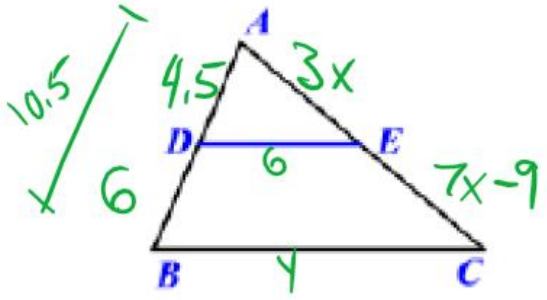
Use the Triangle Proportionality Theorem to solve the following problems.

1. If $\overline{AD}$ is 4.5, $\overline{AE}$ is 3x, $\overline{DB}$ is 6, and $\overline{EC}$ is 7x-9,
   a. What is the value of x?

$$\frac{4.5}{6} = \frac{3x}{7x-9} \qquad 18x = 31.5x - 40.5 \qquad -13.5x = -40.5 \qquad x = 3$$

   b. What is the length of $\overline{BC}$ if $\overline{DE}$ is 6?

$$\frac{4.5}{10.5} = \frac{6}{y} \qquad 63 = 4.5y \qquad 14 = y \qquad \overline{BC} = 14$$



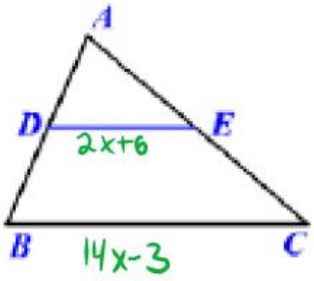
Use the Triangle Midsegment Theorem to solve the following problems.

1. $\overline{DE}$ is a midsegment of ΔABC. Formula 2(midsegment) = parallel side
   a. If $\overline{DE} = 2x + 6$ and $\overline{BC} = 14x - 3$, what is the value of x?

$$2(2x+6) = 14x-3$$
$$4x+12 = 14x-3$$
$$12 = 10x-3$$
$$15 = 10x$$
$$1.5 = x$$

   b. What is the length of $\overline{DE}$?

$$\overline{DE} = 2(1.5)+6 = 9$$



Use the Exterior Angle Theorem to solve the following problems.

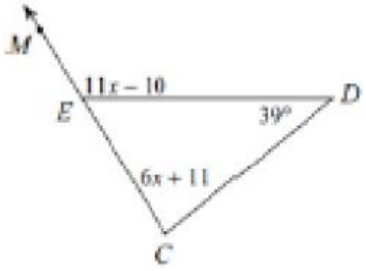
Exterior ∠ = inside ∠1 + inside ∠2

1. What is the value of x in the figure on the right?

$$6x+11+39 = 11x-10$$
$$6x+50 = 11x-10$$
$$50 = 5x-10$$
$$60 = 5x$$
$$12 = x$$

2. What is the measure of $\angle CED$? Linear pairs with ∠MED

$$m\angle MED = 11(12)-10 = 122^\circ$$
$$m\angle CED + 122 = 180$$
$$m\angle CED = 58^\circ$$



Use the Triangle Sum Theorem to solve the following problems.

Interior ∠s of Δ add up to 180°

1. Find the value of each of the missing angles.

x

$$x+39+102 = 180$$
$$x+141 = 180$$
$$x = 39^\circ$$

y

$$39+34+y = 180$$
$$73+y = 180$$
$$y = 107^\circ$$

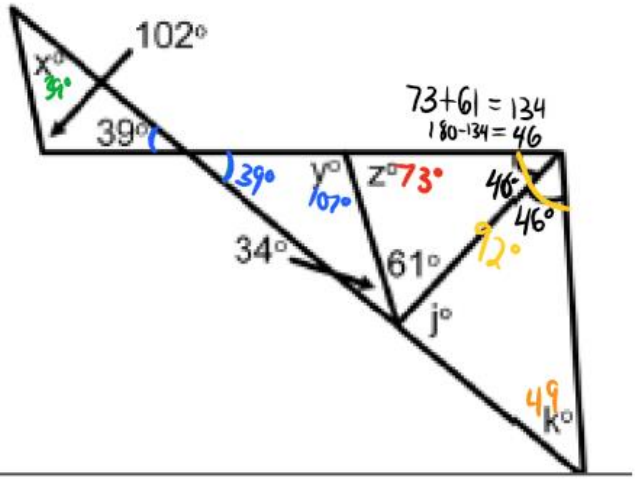
z

y and z are supp.

$$107+z = 180$$
$$z = 73^\circ$$

k

$$39+92+k = 180$$
$$131+k = 180$$
$$k = 49^\circ$$



EOC Review

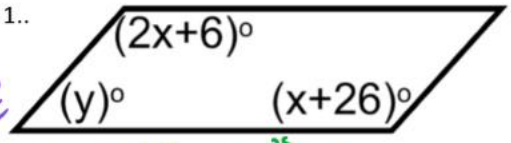
Parallelogram Properties

1. Opp sides ≅
2. Opp ∠'s ≅
3. consecutive ∠'s Supplementary
4. Diagonals bisect each other

Parallelogram Properties

Find the value of x and y in the following parallelograms.

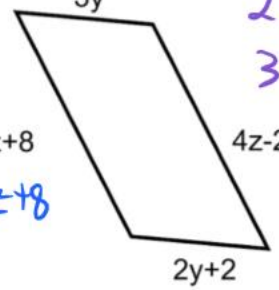
1..



opp ∠'s ≅
$2x+6 = x+26$
$x+6 = 26$
$x = 20$

cons. ∠'s supp.
$y + x + 26 = 180$
$y + (20) + 26 = 180$
$y + 46 = 180$
$y = 134°$

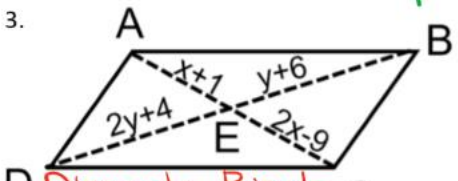
2.



opp. sides ≅
$3y = 2y+2$
$y = 2$

$4z-2 = 2z+8$
$2z-2 = 8$
$2z = 10$
$z = 5$

3.

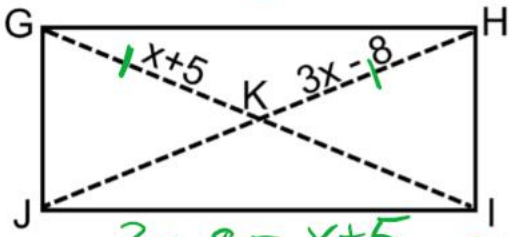


Diagonals Bisect

$2x-9 = x+1$
$x-9 = 1$
$x = 10$

$2y+4 = y+6$
$y+4 = 6$
$y = 2$

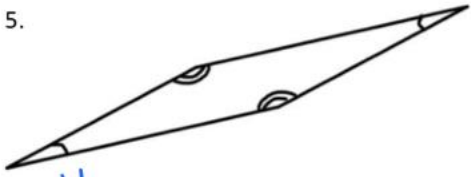
4. $\square GHIJ$ is a rectangle. Find the value of x.



$3x-8 = x+5$
$2x-8 = 5$
$2x = 13$
$x = 6.5$

Could the following quadrilaterals be parallelograms? Provide an explanation of how you know.

5.

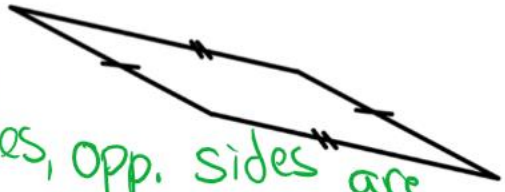


Yes, because opp. angles are congruent.

6.

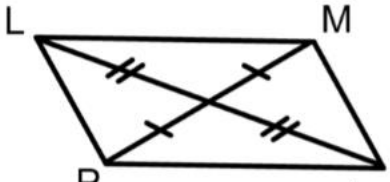
No, opp. sides need to be congruent.

7.



Yes, opp. sides are Congruent

8.



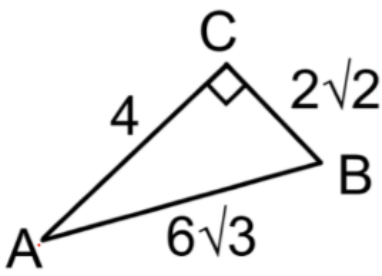
Yes, Diagonals bisect each other

Right Triangle Trigonometry

EOC Review

Identify the basic trigonometric ratios for both acute angles of the following triangles. Answers in simplest radical form.

9.



$\sin(A) = \frac{2\sqrt{23}}{6\sqrt{3}} = \frac{\sqrt{69}}{9}$

$\cos(A) = \frac{4}{6\sqrt{3}} = \frac{2\sqrt{3}}{9}$

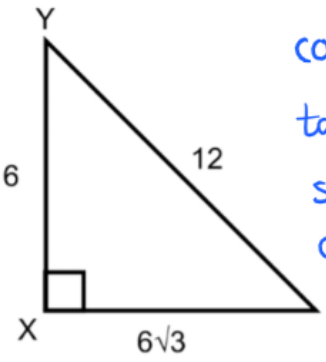
$\tan(A) = \frac{2\sqrt{23}}{4} = \frac{\sqrt{23}}{2}$

$\sin(B) = \frac{4}{6\sqrt{3}} = \frac{2\sqrt{3}}{9}$

$\cos(B) = \frac{2\sqrt{23}}{6\sqrt{3}} = \frac{2\sqrt{3}}{9}$

$\tan(B) = \frac{4}{2\sqrt{23}} = \frac{2\sqrt{23}}{23}$

10.



$\sin(Y) = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$

$\cos(Y) = \frac{6}{12} = \frac{1}{2}$

$\tan(Y) = \frac{6\sqrt{3}}{6} = \sqrt{3}$

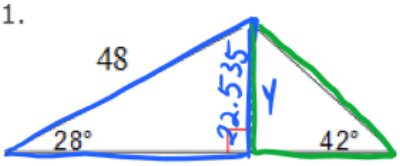
$\sin(Z) = \frac{6}{12} = \frac{1}{2}$

$\cos(Z) = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$

$\tan(Z) = \frac{6}{6\sqrt{3}} = \frac{\sqrt{3}}{3}$

Solve for x.

11.



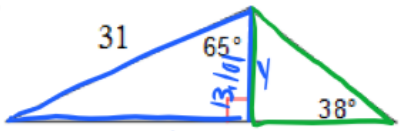
$\sin(28) = \frac{y}{48}$

$y = 48 \cdot \sin(28) = 22.535$

$\tan(42) = \frac{22.535}{x}$

$x = \frac{22.535}{\tan(42)} = 25.027$

12.



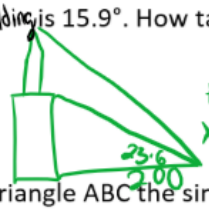
$\cos(65) = \frac{y}{31}$

$y = 31 \cdot \cos(65) = 13.101$

$\tan(38) = \frac{13.101}{x}$

$x = \frac{13.101}{\tan(38)} = 16.769$

13. A communications tower is built on top of a building with the following specifications. From a point 200 meters from the base of the building, the angle of elevation to the top of the tower is 23.6° and the angle of elevation to the top of the building is 15.9°. How tall is the tower?



Tower+Building: $\tan(23.6) = \frac{x}{200}$

$x = 200 \cdot \tan(23.6) = 87.378$

Building: $\tan(15.9) = \frac{y}{200}$

$y = 200 \tan(15.9) = 56.971$

Height of Tower

$87.378 - 56.971 = 30.407$ meters.

14. In triangle ABC the sin(A) = 4/5. Find the following.

opp hyp

$\cos(B) = \frac{4}{5}$

$\tan(A) = \frac{4}{3}$

$m\angle A = \sin^{-1}(4/5) = 53.130$

Missing side

$a^2 + 4^2 = 5^2$

$a^2 + 16 = 25$

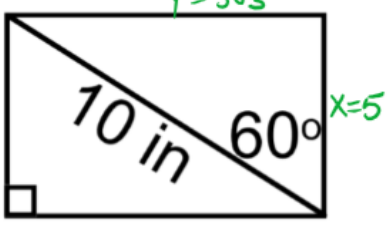
$a^2 = 9$

$a = 3$

Perimeter of ΔABC

Add up all sides: $3+4+5 = 12$

15. Find the perimeter and area of the following rectangle.



$\cos(60) = \frac{x}{10}$

$x = 10 \cdot \cos(60)$

$x = 5$

$\sin(60) = \frac{y}{10}$

$y = 5\sqrt{3}$

Perimeter: $5+5+5\sqrt{3}+5\sqrt{3} = 27.321$

EOC Review

16. What are the steps for construction an inscribed square?

1. Draw a diameter of the circle.
2. Construct a perp. bisector of the diameter.
3. Connect the 4 points on the circle (2 for diameter, 2 for perp. bisect)

17. What construction is also completed when constructing parallel lines?

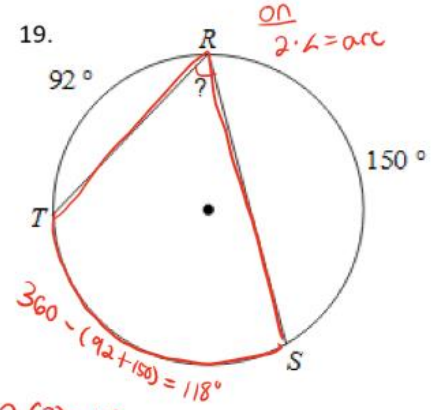
Congruent angles construction

18. What are the steps for constructing a perpendicular bisector?

1. Place the compass center on an endpoint of the line segment and draw an arc over half the distance of the line from the endpoint.
2. Using the same distance draw an arc with the center of the compass on the other end of the segment.
3. Connect the points of intersection of the two arcs.

Circles Properties

19.

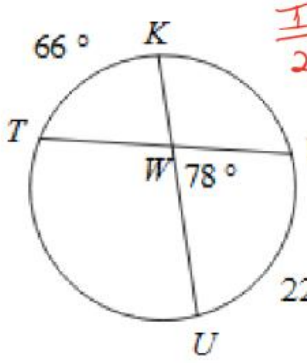


on
$2 \cdot \angle = \text{arc}$

$2(?) = 118$

$? = 59°$

20.



In
$2(\angle) = \hat{1} + \hat{2}$

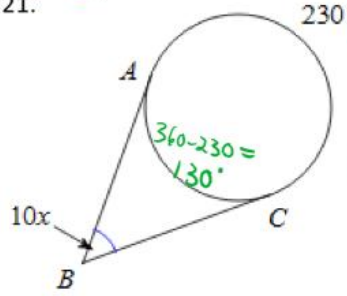
$2(78) = 66 + 22x + 2$

$156 = 68 + 22x$

$88 = 22x$

$x = 4$

21.



Outside
$2(\angle) = \text{Big} - \text{Small}$

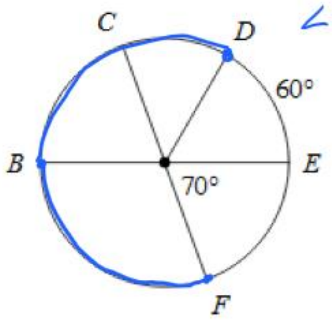
$2(10x) = 230 - 130$

$20x = 100$

$x = 5$

22.

$m\widehat{FBD}$



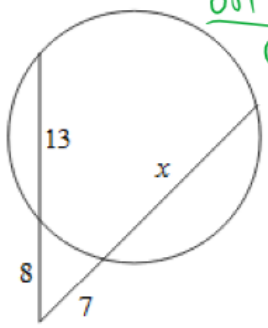
Center
$\angle = \text{arc}$

$m\widehat{FBD} = 360 - (60 + 70)$

$= 230°$

EOC Review

23.



Out Segment

$O(W) = O(W)$

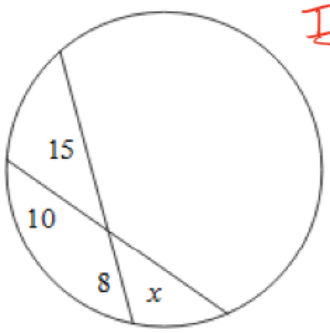
$7(7+x) = 8(8+13)$

$49+7x = 168$

$7x = 119$

$x = 17$

24.



In Segment

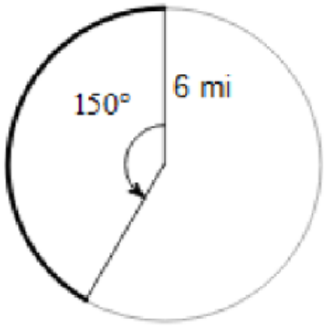
$P \cdot P = P \cdot P$

$15(8) = 10(x)$

$120 = 10x$

$x = 12$

25. Find the arc length.

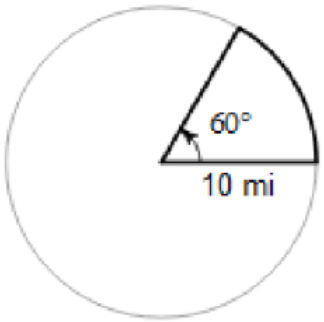


$\frac{150}{360} = \frac{x}{2\pi(6)}$

$360x = 1800\pi$

$x = 5\pi$

26. Find the sector area.



$\frac{60}{360} = \frac{x}{\pi(10)^2}$

$360x = 6000\pi$

$x = \frac{50\pi}{3}$

Derivation of formulas

27. Where does the value of pi come from?

The value of pi comes from dividing the circumference of a circle by its diameter.

28. Describe what Cavalieri's Principle is, and why it works.

29. Describe how the equation for a cone could be derived.

It takes 3 cones to fill a cylinder with equal base area and height. So 3 cones = 1 cylinder

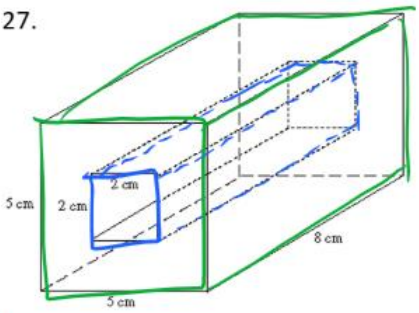
↳ cone = $\frac{1}{3}$ cylinder → $V_{cone} = \frac{1}{3}\pi r^2 h$

30. Where does the equation for the area of a circle come from?

By cutting a circle into infinitely many congruent sectors, then rearranging the sectors to form a rectangle with dimensions, h = radius and $l = \frac{1}{2}C = \pi r$. The area would then be $A = \pi r \cdot r = \pi r^2$. This the area Volume of a circle equation.

EOC Review

27.

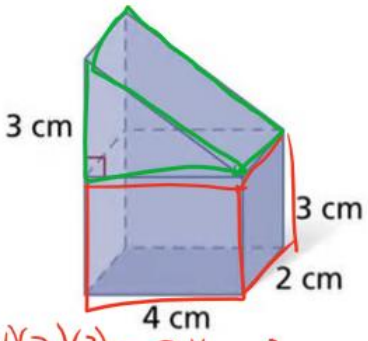


$V_{RP} = 5 \cdot 5 \cdot 8 = 200\ cm^3$

$V_{RP} = 2 \cdot 2 \cdot 8 = 32\ cm^3$

Total volume $= 168\ cm^3$

28.



$V_{RP} = (4)(2)(3) = 24\ cm^3$

$V_{TP} = \frac{1}{2}(4)(3)(2) = 12\ cm^3$

Total volume $= 36\ cm^3$

29. Find the thickness x of the hollow cylinder of height 100 cm if the volume between the inner and outer cylinders is equal to 11000 Pi mm^3 and the outer diameter is 12 mm.

$V_{cylinder} = \pi(6)^2(1000) = 36000\pi\ mm^3$

$-\ 11\,000\pi\ mm^3$

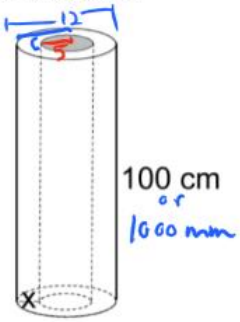
$V_{inside\ cylinder} = 25000\pi\ mm^3$

$25000\pi = \pi(r)^2(1000)$

$25 = r^2$

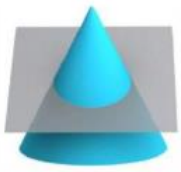
$5 = r$

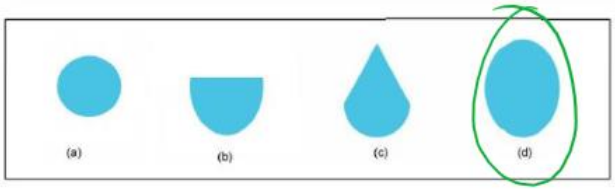
$r_{out\ cylinder} - r_{in\ cylinder} =$ 1 mm



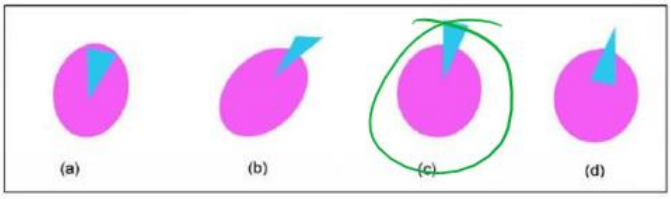
Cross Sections

30.



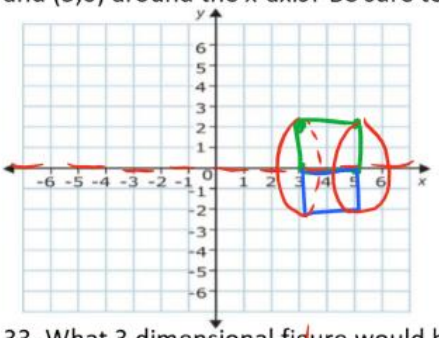


31.



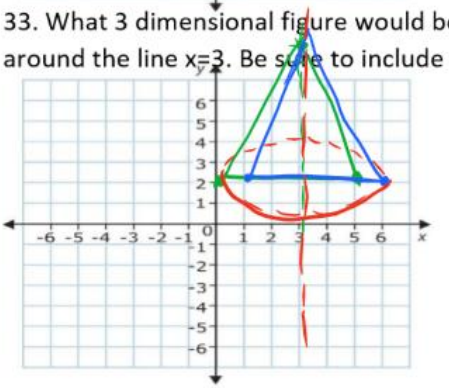
EOC Review

32. What 3 dimensional figure would be created from rotating the 2 dimensional figure with points (3,2), (5,2), (3,0), and (5,0) around the x-axis? Be sure to include radius and height.



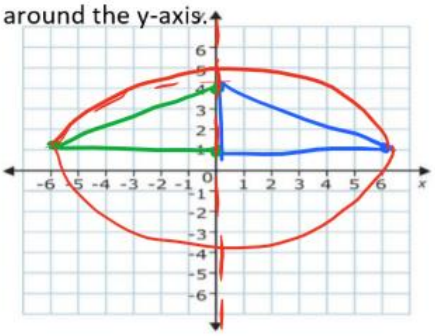
Cylinder with a radius of 2 and a height of 2

33. What 3 dimensional figure would be created from rotating the 2 dimensional figure with points (0,2), (5,2), and (3,9), around the line x=3. Be sure to include radius and height.



A cone with a radius of 3 and height of 7

34. What 3 dimensional figure would be created from rotating the 2 dimensional figure with points (0,4), (0,1), and (6,1), around the y-axis.



A cone with a radius of 6 and height of 3.

<u>Parallel and Perpendicular Lines</u>

35. Write the equation of a line that is parallel to $y = 2x + 9$ and goes through point (8, 20).

original
$m = 2$

New
$m = 2$
$y = 2x + b$
$(20) = 2(8) + b$
$20 = 16 + b$
$4 = b$
$y = 2x + 4$

36. Write the equation of a line that is perpendicular to $5 + y = \frac{1}{4}x + 18$ and goes through point (-6, 12)

original
$5 + y = \frac{1}{4}x + 18$
$y = \frac{1}{4}x + 13$
$m = \frac{1}{4}$

New
$m = -4$
$y = -4x + b$
$(12) = -4(-6) + b$
$12 = 24 + b$
$-12 = b$
$y = -4x - 12$

37. Write the equation of a line that is parallel to $9 + 2y = 4x + 11$ and goes through point (-4, 2)

original
$9 + 2y = 4x + 11$
$2y = 4x + 2$
$y = 2x + 1$
$m = 2$

New
$m = 2$
$y = 2x + b$
$2 = 2(-4) + b$
$2 = -8 + b$
$10 = b$
$y = 2x + 10$

<u>Section Formula</u>

$(x,y) = \left(\frac{ax_2+bx_1}{a+b}, \frac{ay_2+by_1}{a+b}\right)$

EOC Review

38. Line segment AB has endpoints (-4, 1) and (3, -6). What are the coordinates of the point that divides AB in the ratio of 4:3?

(x_1, y_1) (x_2, y_2) (a, b)

$(x,y) = \left(\frac{4(3)+3(-4)}{4+3}, \frac{4(-6)+3(1)}{4+3}\right) = (0,3)$

39. Line segment AB has endpoints (-3, 3) and (0, -6). What are the coordinates of the point that divides AB in the ratio of 1:2?

(x_1, y_1) (x_2, y_2)

$(x,y) = \left(\frac{1(0)+2(-3)}{1+2}, \frac{1(-6)+2(3)}{1+2}\right) = (-2,0)$

40. Line segment AB has endpoints (6, 1) and (-6,1). What are the coordinates of the point that divides AB in the ratio of 3:1?

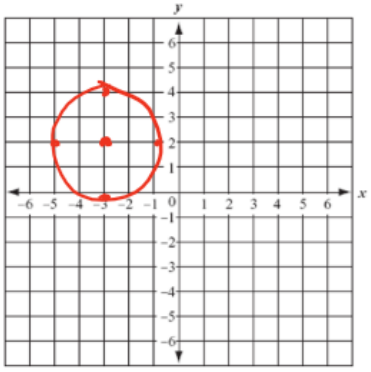
(x_1, y_1) (x_2, y_2) (a, b)

$(x,y) = \left(\frac{3(-6)+1(6)}{3+1}, \frac{3(1)+1(1)}{3+1}\right) = (-3,1)$

Graphing Circles

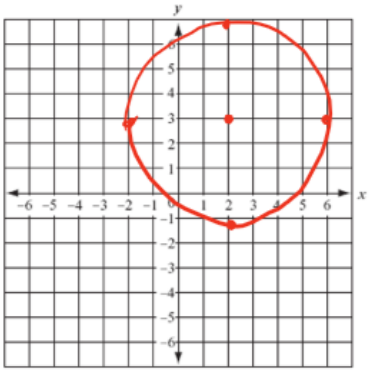
41. $(x+3)^2 + (y-2)^2 = 4$

r= 2 center= (-3,2)



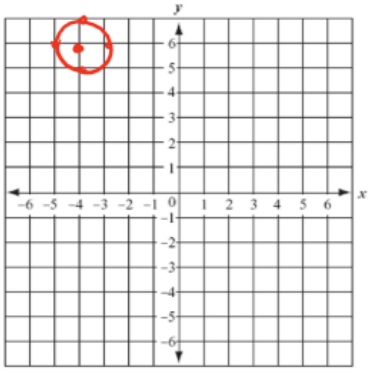
42. $(x-2)^2 + (y-3)^2 = 16$

r= 4 center= (2, 3)



43. $(x+4)^2 + (y-6)^2 = 1$

r= 1 center= (-4,6)



44. $(x+3)^2 + (y-3)^2 = 16$

r= 4 center= (-3,3)