Multiplication Rule

P(A and B) = P(A) * P(B)

If I want to know the probability of one event happening right after another, and these events are independent, we would use the multiplication rule.

EXAMPLE 1: Select two cards from the standard deck of 52 cards with replacement. Find the probability of selecting two kings.

$P(K) = \frac{4}{52}$ $P(K) = \frac{4}{52}$

$P(K \text{ and } K) = \frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2704} = \frac{1}{169}$

EXAMPLE 2: Suppose you toss a coin and then roll a dice. What is the probability of obtaining a tail and then rolling a 5

$P(T) = \frac{1}{2}$ $P(5) = \frac{1}{6}$

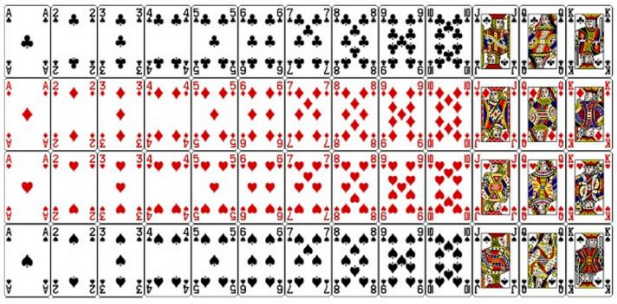
$P(T \text{ and } 5) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

Consider the following two problems:

(1) Select 2 cards from a standard deck of 52 cards with replacement. What is the probability of obtaining two kings?

(2) Select 2 cards from a standard deck of 52 cards without replacement. What is the probability of obtaining two kings?

Do these have the same probability?



No they do not. One is with replacement and one is without.

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

This is called conditional probability. Event B has happened on the condition that Event A has already happened. Event B given Event A notated (B|A).

EXAMPLE 3: Select two cards from the standard deck of 52 cards without replacement. Find the probability of selecting two kings

$P(K) = \frac{4}{52} \quad P(K|K) = \frac{3}{51}$

$P(K \text{ and } K) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$

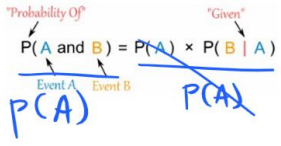
A committee consists of four women and three men. The committee will randomly select two people to attend a conference in Hawaii. Find the probability that both are women

Total = 7

$P(W) = \frac{4}{7} \quad P(W|W) = \frac{3}{6}$

$P(W \text{ and } W) = \frac{4}{7} \cdot \frac{3}{6} = \frac{12}{42} = \frac{2}{7}$

Condtional probability Formula



$P(A)$ $P(A)$

$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

Ex. 1 A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

$P(B \text{ and } W) = 0.34 \quad P(B) = 0.47$

$P(W|B) = \frac{0.34}{0.47} = 0.72$

The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

$P(F \text{ and } Absent) = 0.03 \quad P(F) = 0.2$

$$P(Absent \mid Fri) = \frac{0.03}{0.2} = 0.15$$