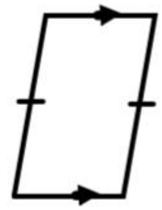
Warm-Up

Will the following quadrilateral always be a parallelogram?

1. 

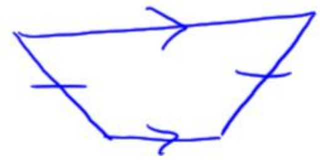
yes

2. 

NO

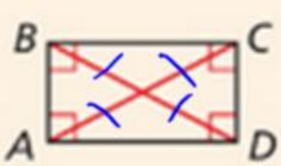
If you chose yes, identify the property. If you chose no, provide a counter example.

1. Both pairs of opp. ∠'s ≅
So it must be a ▱

2. 

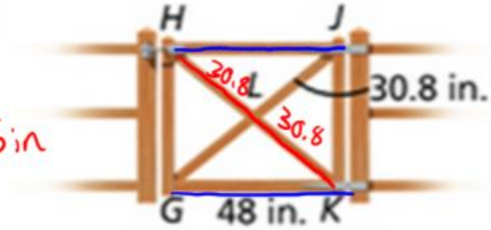
Now that we have SOOOO much information about parallelograms, we need to look at a special parallelogram.

A type of special quadrilateral is a <u>rectangle</u>. A <u>rectangle</u> is a quadrilateral with four right angles.

THEOREM	HYPOTHESIS
If a quadrilateral is a rectangle, then it is a parallelogram. (rect. → ▱)	
If a parallelogram is a rectangle, then its diagonals are congruent. (rect. → diags. ≅)	

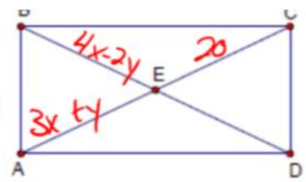
Carpentry The rectangular gate has diagonal braces. Find each length.

1a. HJ = 48 in **1b.** HK = 61.6 in



Practice Problems 1-4

1. In the diagram of rectangle ABCD, diagonals AC and BD intersect at E. If AE = $3x + y$, BE = $4x - 2y$ and CE = 20 , find x and y.



$4x-2y=20 \rightarrow 4x-2y=20$

$2(3x+y=20) \rightarrow +\ 6x+2y=40$

Eliminate y by adding

$10x=60 \rightarrow x=6$

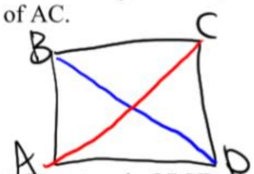
Use x to find y.

$3(6)+y=20$

$18+y=20$

$y=2$

2. In rectangle ABCD, diagonals AC and BD are drawn. If AC = $x^2 + 4x - 23$ and BD = $5x + 33$, find the length of AC.



Diagonals are ≅

$x^2+4x-23=5x+33$

$x^2-x-23=33$

$x^2-x-56=0$ (-56; 7, -8)

$(x+7)(x-8)=0$

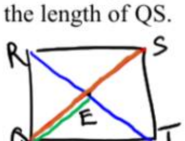
$x+7=0 \quad x-8=0$

$\cancel{x=-7} \quad x=8$ sub in to find $m\overline{AC}$

$m\overline{AC}=(8)^2+4(8)-23$

$m\overline{AC}=73$

3. In rectangle QRST, diagonals QS and RT intersect at E. If QE = $3x - 10$ and QS = $5x - 8$, find the length of QS.



$2(\overline{QE})=QS$

$2(3x-10)=5x-8$

$6x-20=5x-8$

$x-20=-8$

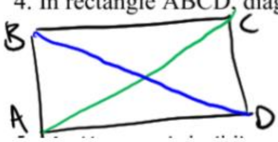
$x=12$

$m\overline{QS}=5(12)-8$

$m\overline{QS}=60-8$

$m\overline{QS}=52$

4. In rectangle ABCD, diagonal AC = 6x-2 and diagonal BD=4x+2. Find the length of AC.



$6x-2=4x+2$

$2x-2=2$

$2x=4$

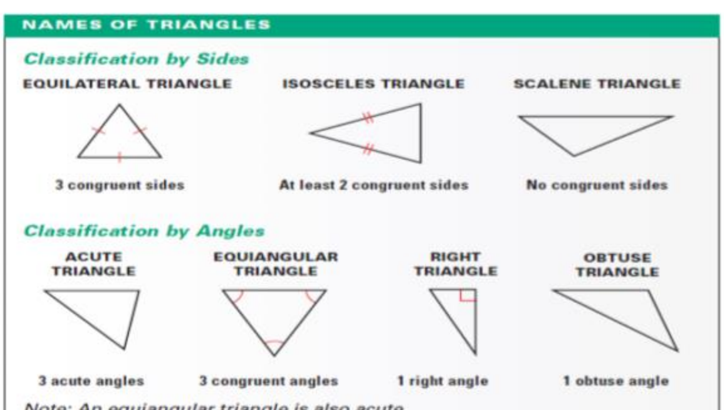
$x=2$

$m\overline{AC}=6(2)-2$

$m\overline{AC}=12-2$

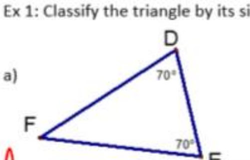
$m\overline{AC}=10$

And now, some more triangle properties



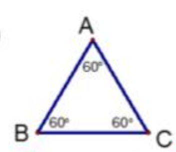
Ex 1: Classify the triangle by its sides and angles.

a)



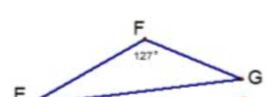
Acute Isosceles

b)



Equiangular, Equilateral

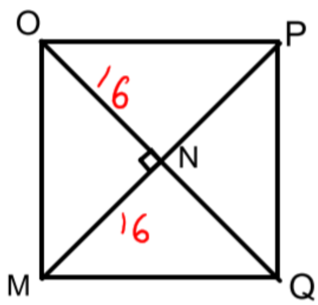
c)



Obtuse Scalene

Ex 2: On worksheet

MN=16, NO=16



a. Explain why ΔMNO is an isosceles right triangle.

It has a right angle and two congruent sides.

b. Identify the hypotenuse and legs of ΔMNO

Hyp → $\overline{OM}$ Legs → $\overline{ON}$ + $\overline{MN}$

Triangle Sum Theorem

Do you remember the triangle sum theorem? What did it say?

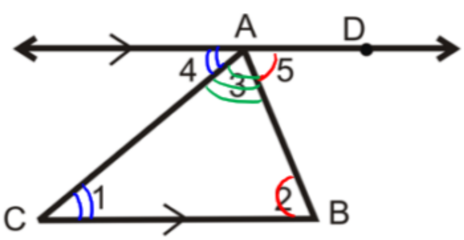
Sum of interior angles of a triangle is 180°

Proof of Triangle Sum Theorem

(Interior Angle Sum Theorem)

Given: ΔABC and $\overleftrightarrow{AD} \| \overleftrightarrow{CB}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



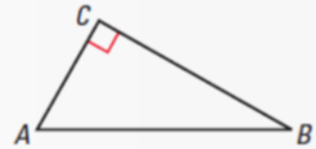
S	R
ΔABC + $\overline{AD} \| \overline{CB}$	Given
$m\angle 1 + m\angle 5 + m\angle 3 = 180°$	Def. of straight angle
$\angle 1 \cong \angle 4$ + $\angle 2 \cong \angle 5$	Alt. Int. ∠'s Theorem
$m\angle 1 = m\angle 4$ + $m\angle 2 = m\angle 5$	Def. of congruence
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$	Substitution

COROLLARY

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are complementary.

$m\angle A + m\angle B = 90°$



corollary – forming a proposition that follows from one already proved.

Ex. 3 On Worksheet

Ex 3: ΔABC is a right triangle. $\angle A$ and $\angle B$ are acute angles. Determine the value of the acute angle in the following examples.

a) $m\angle A = 27°, \angle B =$ 63°

$27 + m\angle B = 90$

$m\angle B = 63°$

b) $m\angle A = 15°, \angle B =$ 75°

$15 + m\angle B = 90$

$m\angle B = 75°$

c) $m\angle A = 45°, \angle B =$ 45°

$45 + m\angle B = 90$

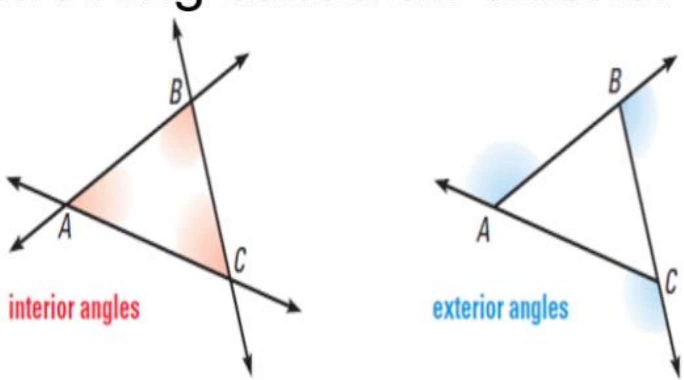
$m\angle B = 45$

d) $m\angle A = 74°, \angle B =$ 16°

$74 + m\angle B = 90$

$m\angle B = 16$

Next we are going to talk about something called an exterior angle.



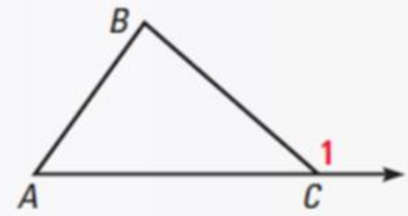
<u>Exterior Angle</u>- the angle between any side of a shape, and a line extended from the next side.

THEOREM

THEOREM 4.2 ***Exterior Angle Theorem***

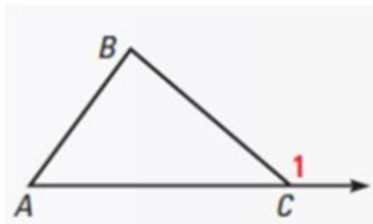
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle A + m\angle B$$



Ex 14: Given: $\angle 1$ is an exterior angle of ΔABC.

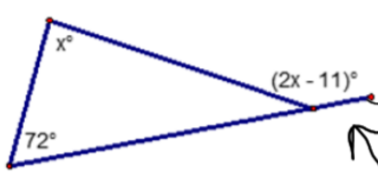
Prove: $m\angle 1 = m\angle A + m\angle B$



Statements	Reasons
1. $\angle 1$ is an exterior angle of ΔABC.	1. Given
2. $\angle ACB$ and $\angle 1$ are a linear pair	2. Definition of Exterior Angle
3. $m\angle ACB + m\angle 1 = 180°$	3. Linear Pairs Conjecture
4. $m\angle A + m\angle B + m\angle ACB = 180°$	4. Triangle Sum Theorem
5. $m\angle ACB + m\angle 1 = m\angle A + m\angle B + m\angle ACB$	5. Transitive Property
6. $m\angle 1 = m\angle A + m\angle B$	6. Inverse Prop. of Addition

Ex 13: Find the value of x. then find the measure of the exterior angle.

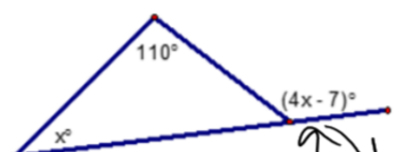
a)



$72 + x = 2x - 11$
$+11 \qquad +11$
$83 + \cancel{x} = 2x$
$-\cancel{x} \qquad -x$
$83 = x$

$2(83) - 11$
$166 - 11$
$155°$

b)



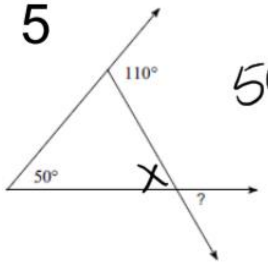
$x + 110 = 4x - 7$
$+7 \qquad +7$
$x + 117 = 4x$
$-x \qquad -x$
$\frac{117}{3} = \frac{3x}{3}$
$39 = x$

$4(39) - 7$
$156 - 7$
$149°$

Practice Problems

5, 7, 13, 14

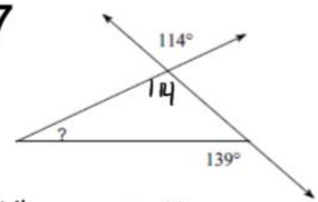
5



$50 + x = 110$

$x = 60°$

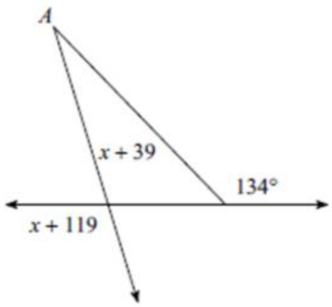
7



$114 + x = 139$

$x = 25°$

13



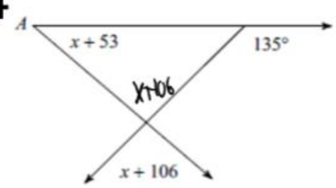
$x + 39 + x + 119 = 134$

$2x + 158 = 134$

$2x = -24$

$x = -12$

14



$x + 53 + x + 106 = 135$

$2x + 159 = 135$

$2x = -14$

$x = -7$

Triangle Theorems

Classify the following triangles by side and angle.

1.  Right Isosceles

2.  Obtuse Scalene

3.  Equilateral

Complete each statement using always, sometimes or never.

An Isosceles triangle is sometimes an equilateral triangle

An Obtuse triangle is Sometimes an isosceles triangle.

An interior angle of a triangle and one of it's adjacent exterior angles are always supplementary

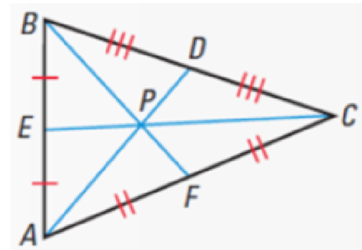
The acute angles of a right triangle are always complementary.

A triangle never has a right angle and obtuse angle

Concurrency of Medians of a Triangle Theorem: The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

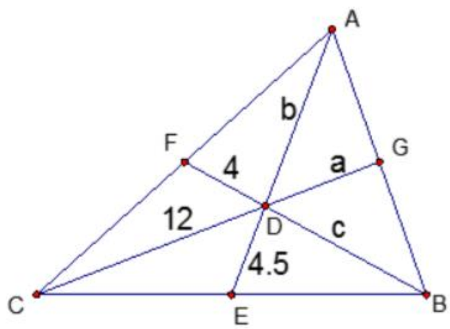
If P is the centroid of ΔABC , then

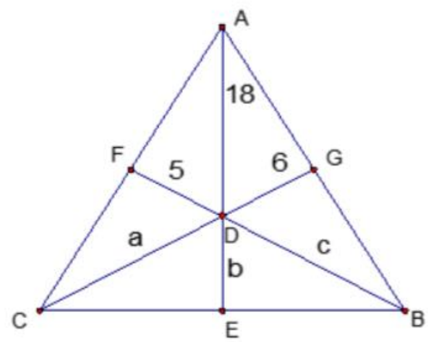
$AP = \frac{2}{3}AD,\ BP = \frac{2}{3}BF,$ and $CP = \frac{2}{3}CE$



The point of concurrency of the medians of a Δ is called the centroid.

Using the relationship with centroids, solve for a, b, and c in the triangle.





$2a = 12$

$a = 6$

$2(4.5) = 9$

$b = 9$

$2(4) = 8$

$2(6) = 12$

$a = 12$

$2(b) = 18$

$b = 9$

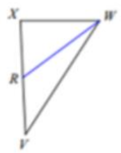
$2(5) = c$

$c = 10$

Practice
Do problems 3 and 4

1 1

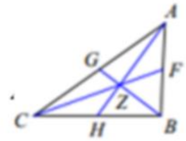
3) Find x if $RV = x$ and $RX = 2x - 4$



$2x-4=x$
$-x \quad -x$
$x-4=0$
$x=4$

3 1

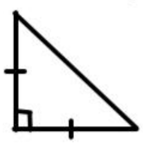
Find x if $AH = -6 + 4x$ and $ZH = x$



$3(x)=-6+4x$
$3x=-6+4x$
$-4x \quad -4x$

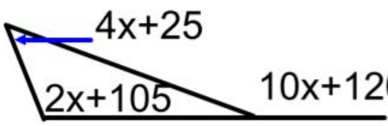
Review

1. Classify the following triangle by angles and sides.



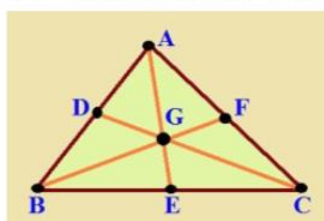
Right Isosceles

2. Determine the value of x.



$4x+25+2x+105=10x+120$
$6x+130=10x+12\cancel{0}$
$-120 \quad -\cancel{120}$
$\cancel{6x}+10=10x$
$-\cancel{6x} \quad -6x$
$\frac{10}{4}=\frac{4x}{4}$
$\boxed{x=2.5}$

3. Determine the length of AE.



2
$m\overline{AG} = 10x$
1
$m\overline{GE} = 2x+9$

$2(2x+9)=10x$
$4\cancel{x}+18=10x$
$-\cancel{4x} \quad -4x$
$\frac{18}{6}=\frac{\cancel{6}x}{\cancel{6}}$
$\boxed{x=3}$